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CONVERGENT AND NON-CONVERGENT SEQUENCES

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Abstract

A given sequence is either convergent or non-convergent and a non-convergent sequence is either divergent oscillatory. Here I mention briefly how to easily identify a given sequence is convergent or divergent or oscillating with their definitions.

Keywords: *Convergent, non-convergent, divergent, oscillating bounded, unbounded, limit points, neighborhoods etc.*

INTRODUCTION

A convergent sequence must be bounded, monotonic and has single limit point where it converges. A non-convergent sequence may be bounded or unbounded and may be monotonic or non-monotonic. A non-convergent sequence may or may not have limit points.
In general there are two types of sentences:-
One, Convergent (cgt.) sequences and next Non-convergent (Non-cgt.) sequences

1- Convergent(cgt.) Sequences

Definition :-A sequence $\{x_n\}$ in R is said to be convergent iff $\exists$ apoint x_0 in R,satisfying the following condition:

$(\forall\in>0)$ $(\exists N\in Z^+)$s.t. $\forall n \geq N \Rightarrow |x_n - x_0| <\in$

i.e., $\forall n \geq N \Rightarrow x_n \in N(x_n, \in)$.

In the case, we say that the sequence $\{x_n\}$ converges to the point x_0 and we write $\lim_{n \to \infty} x_n = x_0$ or $x_x \to x_0$ as $n\to\infty$.

Notes

1- The point x_0 where the sequence $\{x_n\}$ converges is the unique limit point of the Sequence $\{x_n\}$
2- From the defination of cgt.sequences, it follows that: a sequence $\{x_n\}$ converges to x_0 iff every $\in$ -nbd of x_0 contains all, except finite, terms of the sequence $\{x_n\}$.
3- If an $\in$-nbd$n(x_0, \in)$ of x_0 contains all, except finite, terms of the sequence $\{x_n\}$ then the nbd $n(x_0, \in)$ is said to confine the sequence $\{x_n\}$.

Some examples

Ex.1 Let x_n= c, a constaint, $\forall$ n $\in Z^+$. Then $\{x_n\}$=c,c,c,.........is a constant sequence. This constant sequences is cgt. and converges to c i.e., $\lim_{n \to \infty} x_n = c$

Solution

Clearly $\forall\in>0$and $\forall$ n€Z^+, we have $|x_n - c| = |c - c| = |0| = 0 <\in$

Ex.2: Let $x_n=1/n$ $\forall$ $n \in Z^+$, then the sequence $\{x_n\}=\{1/n\}=1,\frac{1}{2},\frac{1}{3},$……..iscgt.and converges to 0. i.e., $x_n = \lim_{n\to\infty} 1/n = 0$

Solution

To show $\{x_n\}=\{1/n\}$ converges to 0, let $\in > 0$ be given.

Now ,$|1/n - 0| < \in \Leftrightarrow 1/n < \in \Leftrightarrow n > 1/\in$.

Which shows that $(\forall \in > 0)(\exists N \in Z^+$ with $N > 1/\in)$ s.t.

$\forall n \geq N \Rightarrow 1/n \leq 1/N < \in$

$$i.e., \forall n \geq N \Rightarrow |1/n - 0| < \in$$

$\therefore x_n = \lim_{n\to\infty} 1/n = 0$. i.e., $\{x_n\} = \{1/n\}$ converges to 0.

Ex.3:. The sequence $\{x_n\}=\{1+\frac{(-1)^n}{2n^2}\}$ converges to 1.

Solution

To show the given sequence $\{x_n\}$ converges to 1, Let $\in > 0$ be given,

Now, $|x_n - 1| < \in \Leftrightarrow \left|\frac{(-1)^n}{2n^2}\right| < \in \Leftrightarrow \frac{1}{2n^2} < \in \Leftrightarrow n > \frac{1}{\sqrt{2\in}}$

Which show that ,$(\forall \in > 0)$ $(\exists N \in Z^+$with $N > 1/\sqrt{2\in})$ s.t.

$\forall n \geq N \Rightarrow 1/2n^2 \leq 1/2N^2 < \in$

$$\text{i.e., } \forall n \geq N \Rightarrow |x_n - 1| < \in$$

$\therefore x_n = \lim_{n\to\infty} 1$. i.e., the sequence $\{x_n\}$ convergent to 1.

2- Non-Convergent (Non-cgt.) sequences:-

Definition

A sequence which is not convergent is called a non-convergent sequence.

Non- convergent sequences are classified as under-

i. Divergent(Dgt) sequences-

a) Divergent to $+\infty$and b) Divergent to $-\infty$.

ii. Oscillatory sequences-

a) Finitely Oscillatory and b) Infinitely Oscillatory.

i. Divergent sequences :-Divergent sequences are unbounded above (right) or unbounded below (left) and they have no limit points.

a) Divergent to + ∞ sequences:- A sequence $\{x_n\}$ in R is said to be diverges to $+\infty$iff $\forall k > 0$ (however large) $\exists N \in Z^+$s.t.$\forall n>,N \Rightarrow x_n > k$

In this case we write:-$x_n=+\infty$ or $\lim_{n\to\infty} x_n \to +\infty$ as $n \to \infty$ or simply$x_n \to +\infty$

Note:- Clearly, A sequence which diverges to$+\infty$ is unbounded above (right) and is monotonic increasing.

b) Divergent to $-\infty$sequences:- A sequence $\{x_n\}$ in R is said to be diverges to $-\infty$ iff $\forall k > 0$ (however large) $\exists N \in Z^+$ s.t.$\forall n>,N \Rightarrow x_n < -k$

In this case we write:-$x_n = -\infty$ or $\lim_{n\to\infty} x_n \to -\infty$ as $n \to \infty$ or simply $x_n \to -\infty$

Note:-Clearly, A sequence which diverges to $-\infty$ is unbounded below (left) and is

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monotonicdecreasing.

For example:-

1) The sequence : {n,{n^2},{n^2}, {2^n},{$\frac{n^3+1}{n+1}$} etc. diverges to$+\infty$.
2) The sequence :{-n,{-n^2},{2n-n^2},{$log\frac{1}{n}$} etc. diverges to$-\infty$.

ii. Oscillatory sequences:- A sequence which of neither convergent nor divergent is

said to be an oscillatory sequence. Oscillatory sequences are bdd orunbdd according as they are finitely oscillatory orinfinitely oscillatory.

Obviously,anoscillatory sequence is non-monotonic.

1. Finitely oscillatory sequences:- A finitely oscillatory sequences is bdd and it has at least 2 limit points.
2. Infinitely oscillatory sequences:- An infinitely oscillatory sequences isunbdd (above or below or both) and it may or may not have limit points.

For example:-
Ex.1. The sequence
{x_n}={$(-1)^{n+1}$} = 1,-1,1, -1,……..oscillates finitely. It is bdd and
has 2 limit point 1 and -1
Ex.2. The sequence {x_n}=1,2,3,1,2,3,1,2,3,……oscillates finitely. It is bdd and
has 3 limit points 1,2 and 3.
Ex.3. The sequence {x_n}=1, $\frac{1}{2}$,2, $\frac{1}{3}$,3,$\frac{1}{4}$, 4,…… oscillates infinitely. It is unbounded
(above) and has a limit points o.
Ex.4. The sequence {x_n}=-1, -$\frac{1}{2}$, -2, $-\frac{1}{3}$,-3, $-\frac{1}{4}$, -4,……oscillates infinitely. It is
unbounded(below) and has limit points o.
Ex.5. The sequence {x_n}={$(-1)^n$. n}= -1,2,-3,4,-5, 6,……… oscillates infinitely. It is
unbounded (both) and has no limit point.
The table, below, shows the diagnosis of cgt. anddgt. of real sequences.

Monotoneness → Boundedness ↓	Monotonic	Non-Monotonic
Bounded	Convergent	Finitely Oscillatory
Unbounded	Divergent	Infinitely Oscillatory

CONCLUDING REMARK

From above we obtain the conclusion that a monotonic and bounded sequence is always convergent, a monotonic and unbounded sequence is divergent. Similarly; a non-monotonic and bounded sequence is finitely oscillatory, a non-monotonic and unbounded sequence is infinitely oscillatory.

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