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METRIC SPACES AND APPLICATIONS OF METRIC

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Abstract

Study of metric space is closely related to topology or topological space. In this paper we study the definition of metric, metric space and some examples. Also we learn how the metric is used to define the open set and closed set in metric space. This paper also explore some applications of metric in transport, air transport, machine learning and computer science.

Keywords*: Metric, Metric space, open set, closed set, metric learning, deep metric learning etc*

INTRODUCTION

Topology is branches of mathematics and topological space is a set of points, along with a set of neighborhoods for each point, satisfying a set of axioms relating points and neighborhoods. In Mathematics, Topology has an importance due to their huge application in various field. The Metric Space is closely related to the Topology in that study of metric Space concerned itself also with the set of points and limit points based on a function which gives a distance. The metric space has been working for decade in various applications like internet searching, image classification, protein classification etc. The metric space is a set where a notion of distance (called a metric) between points or elements of the set is defined. Every metric space is a topological space in a natural manner. In metric space, every terms like continuity, convergent , divergent, connectedness, compactness are define in terms of open set and closed set so to understand the concept of open set and closed set are very important. Deep metric learning and embedding metric are based on metrics which played vital role in machine learning and computer science. Also there are so many application of metric not only in basic science but also in our daily life.

METRIC AND METRIC SPACE

Given a set X, a function d : X × X → R is a metric (or distance function) on X if for all x, y ∈ X we have the following three :

i) Positivity: d(x, y) ≥ 0 $\forall x. y \in X$ and d(x, y) = 0 iff x = y.

ii) Symmetric: d(x, y) = d(y, x) $\forall x. y \in X$

iii) The triangle inequality: for all x, y, z ∈ X d(x, z) + d(z, y) ≤ d(x, y).

Metric space

A nonempty set X with a distance function or metric d is called a metric space.The metric space is denoted by (X, d).

1. **Examples of Metric:**

- **The trivial metric or discrete metric :**

  Let X be a set, and d: X × X→ R be the function that maps

  d(x; y) = 1 if x ≠ y, and d(x; x) = 0.

- **The Usual Metric:**

  The usual Distance function on real line is a metric .The Usual Metric R with the metric d(x; y) = lx-yl is metric space.

- **The Euclidean Metric in $\mathbb{R}^2$**

  The Euclidean Metric in R^2 is usual distance function in plane, is defined as

  $$d((x_1, y_1); (x_2, y_2)) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$$

  In general we can define the Euclidean Metric, R^n with the standard distance formula:

  $$d(x, y) = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}$$

- **Taxicab Metric**:

  The name of Taxicab metric was given by Americans and it is a form of nonEuclidean metrics of distance. It is not a shortest distance between given two point. It is used to find the distance in the study of ideal city where all roads are running horizontal or vertical. In such kind of city it not possible to travel from one place to another place by using the normal Euclidean distance so it is modified.

  The Taxicab Metric In two-dimensional space, a taxicab metric that meets the properties of the metric and consists of the set of numbers described by real numbers is defined by the following formula:

  $$d(x,y)=|x_1-y_1|+|x_2-y_2|$$

  The Taxicab Metric In n-dimensional space:

  We can describe the taxicab metric in $\mathbb{R}^{\boldsymbol{n}}$ as follows:

  $$d(x,y)=\sum_{i=1}^{n} | xi - yi |$$

- **The Supremum or maximum Metric:**

  In two-dimensional space or Plane, a supremum metric is defined as

  $$d_\infty(x, y) = \max\{| x_1 - y_1 |, | x_2 - y_2 |\}$$

  In three-dimensional space, a supremum metric is defined as

  $$d_\infty(x, y) = \max\{| x_1 - y_1 |, | x_2 - y_2 |, | x_3 - y_3 | \}$$

  In n-dimensional space, a supremum metric is defined as

  $$d_\infty(x, y) = \max\{| x_i - y_i | / 1 \le i \le n\}$$

2. **Application of Metric:**

   In metric space, the open ball or open sphere is defined by using metric which has an importance in defining limit point, interior point, boundary points of set, convergence of sequences, limit and continuity of function defined on the metric space, Compactness, Connectedness etc.

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Open sphere or Open ball:

If (X, d) is a metric space and x ∈ X. The open sphere with centre x and radius r, is the subset of X given by

Sr (x) = $\{y \in X$: d$(x, y) < r, r > 0\}$

Examples:

- In usual metric space R the open sphere Sr (x) is the open interval (x−r,x+r)
- In discrete metric space X. The open sphere Sr (x) = {x} if 0 < r ≤ 1
  Sr (x) = X if $x > 1$
- In $\mathbb{R}^2$ with Euclidean metric , The open Sphere

Closed sphere or Closed ball:

If (X, d) be a metric space and x ∈ X. The closed sphere with centre x and radius r, is . the subset of X given by

Sr [x] = $\{y \in X$: $(x, y) \leq r, r > 0\}$

Examples:

- In usual metric space R the closed sphere Sr[x] is the closed interval [x − r,x + r].
- In discrete metric space X. The closed sphere Sr[x] ={x} if 0< x < 1
  Sr [x]= X if x ≥

Open set:

A subset A of a metric space (X, d) is called open if, given any point x in A, there exists a real number r > 0 such that, Sr (x)⊂ X .

Equivalently, A is open if every point in A has a neighborhood contained in A.

1. In usual metric space R
- Open interval is an open set.
- R is an open set.
- (a, b] is not an open set.
- The sets ℕ, ℤ and ℚ are not open.
- The set of all irrational numbers is not an open set.
- {x} is not open.
- $\{\frac{1}{n}$ / n∈ $N\}$ is not open.
2. In the discrete metric space X, every set is an open set.

Result

1. In metric space (X, d) the empty set and whole space X are open sets.
- Each open sphere is an open set.
- Arbitrary union of open sets in X is open.
- Finite intersection of open sets in X is open.
- Arbitrary intersection of open sets need not be open.
  If A_n= { (− 1 /n , 1/ n) / n∈ N } then ∩ A_n= { 0 }which is not open.

Closed Set:

A subset A of a metric space (X, d) is called Closed if its complement X−A is open.

Examples:

1. In usual metric space R
- Closed interval is closed set.
- R is closed set.
- [a, b] is closed set.
- The sets $\mathbb{N}$, $\mathbb{Z}$ are closed.
- $\mathbb{Q}$ is not closed
- The set of all irrational numbers is not closed set.
- {x} is closed
- $\{1/n \ / \ n \in N\}$ is not closed
2. In the discrete metric space X, every subset of X is closed set.

Result: In metric space (X, d),

- The empty set and whole space X are closed sets.
- Each closed sphere is closed set
- Finite union of closed sets in X is closed.
- Arbitrary intersection of closed sets in X is closed.
- Arbitrary union of closed sets need not be close.
- If $An = \{ [1/n, 2] \ / \ n \in N \}$ then $\cup An = (0,2]$
- Every singleton set is closed.
- Every finite set in X is closed.

Application of metric in Transport:

The streets of big cities and motorways built in the nineteenth century often had a form of regularly intersecting lines at right angles with rectangular surfaces impassable area of buildings and agricultural land so in such a condition, it was not possible to reach the destination using the shortest route. In two-dimensional space, a taxicab metric is used that meets the properties of the metric and consists of the set of numbers described by real numbers $\mathbb{R}$ is defined by the following formula:

$$d(x,y)=|x_1-y_1|+|x_2-y_2| \quad \text{where } x=(x_1,x_2), \quad y=(y_1,y_2)\in \mathbb{R}^2$$

The journey from departure cities to the destination cannot always be traveled in the shortest way. Of course, the longer path may be better in another respect, for example, quality. There may be several possible but different routes (roads) of the journey, of which not all will have identical lengths or there may be several routes with equal distances. We can consider these problems using the theory of metric spaces for strictly defined of metrics tailored to both types of transport means in one, two, three-dimensional spaces as well as those dependent on the available transport routes. The distances described by a non-classical transport metric between towns P, S, Q lying on the solid of the Earth's ellipsoid and separated by several thousand kilometers from each other. The shortest land route between the P, S and Q places will not be rectilinear, only curvilinear taking into account the curvature of the Earth's ellipsoid. The shortest route between the towns of P, S, Q described by Euclidean metric could be only achieved by air.

Metric Learning:

Metric learning is based on a distance or metric that aims to establish similarity or dissimilarity between objects. The goal of metric learning is to reduce the distance between similar objects or closer the similar objects and to increase the distance between dissimilar objects. The metric learning is based directly on metric or distance that aims to establish the similarity and dissimilarities between the object or image. Metric learning is failed in face recognition and face verification if the faces of same person represented in different poses and expression. For this we need deep metric learning.

Deep metric learning uses neural Network to learn discriminative features from the objects or images and then compute the metric. Deep metric learning is use for the task of face or image verification, face recognition, image classification, anomaly detection etc . There are two loss function, Contrastive loss and Triplet loss which are widely used for deep metric learning. Contrastive loss consists of two identical sub networks that share the same sets of parameters and learn by calculating metric highest level feature encoding of each subnetwork with distinct input. Triplet loss consists of three identical subnetworks that share the same parameter. It is required the distance between the anchor sample and the positive sample to be smaller than the distance between the anchor and negative sample.

The Embedding of metric space has created the interest of several communities like researchers in the networking community as well as researchers in computer science, Mathematicians etc. In recent time embeddings of metric have played a vital role in computer science for evolution of algorithm.Embeddings are also applicable in many areas including computational biology, computer vision, networking and statistics.

CONCLUSION

In this paper we studied the definition of metric, metric space and its examples. Also we saw how open set and closed set are defined in terms of metric and many examples of open and closed sets.This paper shown that how the metric is used in daily life for road transport and air transport, How the metric is applied in metric learning and deep metric learning for image and face recognition and verification.

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