

EXPLORING TEACHERS' PEDAGOGICAL CONTENT KNOWLEDGE IN TEACHING SIMPLE FRACTIONS IN PRIMARY SCHOOLS

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Abstract

The study explored primary school teachers' pedagogical content knowledge of simple fractions in one municipality in the Upper West Region of Ghana. The sequential explanatory mixed method design was employed to explain the results of the quantitative data in much more detail of the phenomenon. The simple random sampling technique was used to sample 150 (95 males and 55 females) participants from both urban and rural schools for the quantitative phase of the study. Thereafter, a sample of 12 respondents (7 males and 5 females) was purposefully sampled for the qualitative phase of the study. A questionnaire, a semi-structured interview, and an observation checklist were used to collect both quantitative and qualitative data. The quantitative data from the questionnaire were analyzed using descriptive statistics (means, standard deviations, and percentages), whereas the qualitative data were analysed thematically. Whereas findings from the questionnaire data showed that the teachers perceived themselves to have pedagogical content knowledge, the interview and observation data proved otherwise. The findings of the quantitative analysis revealed that the teachers generally exhibited limited pedagogical content knowledge of simple fractions. Specifically, the observation checklist revealed that teachers used traditional direct lecture approaches. It was, therefore, recommended that in-service training, workshops and continuous professional development be organized for teachers' to increase their knowledge of and practice in teaching simple fractions.

Keywords: Content Knowledge, Pedagogical Knowledge, Simple Fractions and Mathematical Knowledge

INTRODUCTION

In Ghana, primary mathematics education is governed by a prescribed mathematics curriculum presented in the form of syllabus. The mathematics curriculum provides a detailed prescription of what mathematics content to be taught at each level. It includes the suggested activities that teachers can use to teach for understanding. The curriculum content domain of mathematics for primary education includes but not limited to numbers and investigation with numbers, geometry, estimation and measurement, algebra, statistics and probability (Ministry of Education [MOE], 2012).

Numbers, as one of the principal concepts of Ghanaian primary school mathematics education, covers reading and writing of numerals, number bases (ten, two and five), the four basic operations on number (addition, subtraction, division, and multiplication), ratio, proportion, percentages, fractions, and integers (MOE, 2012). Among the five content domains in the

primary school mathematics curriculum, numbers stands out as the most unique and ultimate pinnacle around which the others revolve.

In the Ghanaian primary school mathematics curriculum, fractions play a central role in the learning of mathematics at the lower primary levels (Basic 1 to 3). The knowledge of fractions has numerous practical applications in human life. Apart from the use of fractional notations in algebra, geometry, and calculus among others, fractions have everyday applications in sharing and measurement problems (Sonnabend, 2010). Without fractions, not all division problems involving integers with non-zero divisors will be possible. Research evidence (e.g., Torbeyns, Schneider, Xin, & Siegler, 2014) suggests understanding fractions relates to pupils' overall mathematics achievement.

MATHEMATICAL KNOWLEDGE FOR TEACHING

Teaching fractions for understanding as emphasized in the Ghanaian curriculum requires teachers who are knowledgeable in the content domain. Teachers' knowledge about the content has been an important area of research in teacher cognition since the 1980s (Dorothy, 2015). Shulman (1986) identified three content domains that are critical for effective teaching. According to Shulman (1986), to successfully teach, the teacher must have: (a) a clear understanding of the subject matter to be taught (content knowledge), (b) a repertoire of accurate teaching presentation strategies that make content accessible to learners (pedagogical knowledge), and (c) a good knowledge of learners conceptions and explanations that clarifies ideas. The teacher must also have a good knowledge of manipulative materials and how to integrate them into an instructional process. All these domains are equally emphasized for effective teaching.

In the content knowledge of fractions the teacher must have basic knowledge and skills required to react to pupils' ideas on fractions (Schifter, 2001), clear understanding of basic concepts on fractions: fraction as a ratio or division of two numbers, numerator, denominator, part-whole relations, and their unique properties, the various representations of fraction and their explanations at the preparatory level (Torbeyns, et al. 2014; Samková, 2018). It also includes paying attention to units as it helps in computational procedures (Schifter, 2001). This theoretical position suggests that for teachers to teach fractions effectively, they must understand what they teach at the level of the learner. Understanding fractions at any level requires a deeper understanding of numbers than ordinarily gained through experience with whole numbers (Torbeyns, Schneider, Xin, & Siegler, 2014).

PCK according to Shulman (1986), comprises of knowledge of the specific subject matter, knowledge of instructional strategies, knowledge of learners' conceptions, and understanding of specific learning difficulties. Teaching fractions includes properly deciding on the parts and wholes of fractions, interpreting and making all fractions correctly visible (Samková, 2018), having deep conceptual connections relevant for teaching of fractions (Schifter, 2001), and the skills for creating opportunities to make learning fractions meaningful. This is because the quality of specific learning opportunities teachers provide, affects pupils' learning and performance (Hattie, 2009).

Shulman's theoretical conception opens a new window of knowledge needed for teaching primary school mathematics (Ball, Thames, & Phelps, 2008). While much attention is focused on studies of mathematics knowledge necessary for primary school teachers to teach mathematics, less attention has been paid to research on specific mathematics content domains such as fractions. The knowledge and behaviour exhibited by teachers determines the extent of their effectiveness in the classroom and, ultimately, influence their pupils' achievement (James, 2007). Cognizance of the influence of teachers' knowledge on pupils' performance, the government of Ghana, in conjunction with her development partners, such as educational stakeholders, local non-governmental organizations, United Nations International Children's Emergency Fund, Japan International Cooperation Agency, United States Agency for International Development, and British Aid stepped up their efforts in enhancing the professional development of teachers through in-service training programmes. Although this helped to update teachers' pedagogical content knowledge and skills for effective classroom practice (MOE & UNESCO, 2012), its impact on specific content domains remains unexamined.

STATEMENT OF THE PROBLEM

Despite the high recognition and importance attached to the teaching and learning of fractions, many primary school teachers and pupils still perceive fractions as difficult and sophisticated to teach and learn respectively (Fazio & Siegler, 2011). This is because fractions are quite different from whole numbers. In fractions, pupils must necessarily understand parts and wholes. For example, in the fraction $1/3$, 1 is the part, 3 the whole and the symbolization called one-third in contrast to whole numbers. That is, the expression denotes that 'a' whole has been divided into 'b' equal parts and each part is represented by 'a'. Beyond this basic symbolic understanding, pupils are required to solve problems on fractions and relate them to real-life contexts, which is a challenge to most pupils in primary school. There have been series of discussions and reports on pupils' poor performance in fractions (Mensah, Okyere, & Kuranchie, 2013). However, previous studies (Hurrell, 2013; Hattie, 2009) reveal that pupils learn what they are taught. Consequently, studies have explored teachers' pedagogical content knowledge in an attempt to address pupils' achievement challenges. For example, researchers have explored PCK in teaching Quadratic Functions (Sibuyi, 2012), pre-service teachers' pedagogical content knowledge and its implication for teaching (Marshman & Poeter, 2013), and the mathematical content knowledge for teaching elementary mathematics (Olanoff, Lo & Tobias, 2014). However, studies that examined teachers' pedagogical content knowledge on fractions in Ghana is scarce. This study, therefore, examined primary school teachers' pedagogical content knowledge in teaching simple fractions in one municipality in the Upper West Region of Ghana.

RESEARCH QUESTIONS

The study was guided by the following questions:

1. What is the content knowledge base of primary school teachers for the teaching of simple fractions?
2. What are primary school teachers' perceived pedagogical practices of teaching simple fractions?
3. What are primary school teachers' knowledge base on pupils' misconceptions?
4. To what extent do primary school teachers' perceived pedagogical practice reflect their class practice of teaching simple fractions?

METHODOLOGY

Research Design

The study utilized a sequential explanatory mixed method research design to explore teachers' pedagogical content knowledge of simple fractions in primary schools. The design is appropriate for the study because of its intensive probing and ability to ensure a comprehensive understanding of the phenomenon under study (Creswell, 2012). This enabled a quantitative sequential study followed by a qualitative exploration of participants' perspectives to triangulate the results generated from the quantitative phase.

Study Area

The study was conducted in one municipality in the Upper West Region of Ghana. The people in the municipality are predominantly subsistence farmers, business persons, and civil servants from all parts of the country due to labour mobility. The urban communities are linguistically heterogeneous whilst the rural ones are linguistically homogeneous. Approximately two-thirds of the population resides in urban localities.

The Upper West Region shares external borders with Burkina Faso to the North and West and internal borders with the Savanna Region to the South and the Upper East Region to the East. It covers a geographical area of 18,476 km² and segmented into 11 administrative districts with Wa as the regional capital (Ghana Statistical Service, 2013). Less than 40.0% of communities in the region have access to primary school within the locality and enrollment in the schools is relatively low.

Participants and context

A simple random sampling technique was used to select 150 (95 males, 55 females) primary school teachers from both urban and rural school communities for the quantitative phase of the study. Out of the 150 participants, 12 (7 males, 5 females) were purposefully sampled for the qualitative phase of the study. Primary school teachers were studied because foundation concepts that serve as the building blocks for real understanding of fractions are developed at that level.

Fractions are introduced early in the learning trajectory in most mathematics curricula (e.g., Britain, USA, Korea, and Israel) worldwide, in recognition of the crucial role fractions play in everyday life in the mathematical growth of a person. In Ghana, fractions are introduced in the mathematics curriculum in Primary 2 at age seven (7) and continues throughout the primary level and beyond (MoE, 2012). The curriculum content of fractions for lower primary (Primary 2 and Primary 3) covers the identification of fractions, symbols of fractions, names of fractions, comparing fractions, locating fractions on a number line and simple addition of like fractions. At the upper primary level (Primary 4 to 6), the content covers different names of fractions, comparing and ordering fractions, converting fractions to decimals and percentages, and operations on fractions (MoE, 2012). At each level in the school ladder, the curriculum delineates the breadth and depth of fractions to be taught, how they should be taught and how they are assessed. Since

understanding fractions is central to pupils achievement in mathematics (Torbeyns, Schneider, Xin, & Siegler, 2014; Samková, 2018), various teaching and learning activities and materials appropriate to the cognitive level of the child are suggested in the curriculum in the bid to encourage teaching for understanding.

Research Instruments

Three instruments namely a Likert-scale type questionnaire, semi-structured interview, and an observation checklist served as data collection tools. The quantitative data were collected through a structured questionnaire and an observation checklist while qualitative data were collected using interviews.

Questionnaire

The questionnaire has four main sections, A to E. Section ‘A’ seeks for participants’ biodata. Sections ‘B’ to ‘E’ are made up of 30 Likert scale-type items on a 4-point scale “strongly disagree (1)”, “disagree (2)”, “agree (3)”, and “strongly agree (4)”. There are 16 items in Section ‘B’ structured to explore teachers’ content knowledge of simple fractions and eight items in Section ‘C’ designed to examine teachers’ perceived pedagogical knowledge on simple fractions. Section ‘D’ also contained six items on teachers’ perceived pedagogical practices on teaching simple fractions. The questionnaires was administered by the author in the participants’ schools. Participants consented to their participation in the study and willingly responded to the questionnaire items and handed them over to the researcher. A total of 150 questionnaires were administered and retrieved from participants’ thereby attaining a 100% return rate. *Semi-Structured Interview.* The semi-structured interview was used to explore teachers’ content knowledge, perceived pedagogical knowledge, and perceived pedagogical practice.

To gain an in-depth understanding of participants’ pedagogical content knowledge, the 12 teachers were interviewed on pupils’ representation of $1/5$ and $2/4$ using the set and the length models respectively as in Figures 1 and 2. Each participant responded to questions i, ii, and iii in both the set and length models.

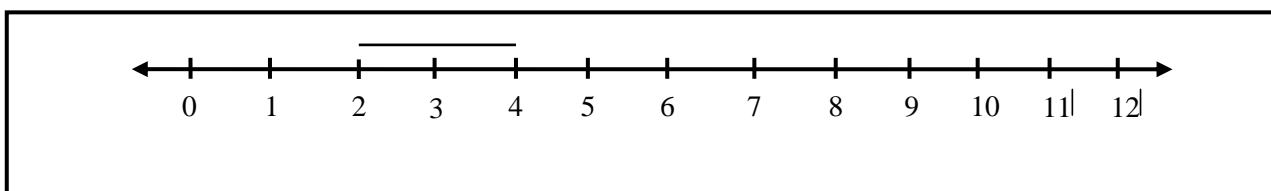
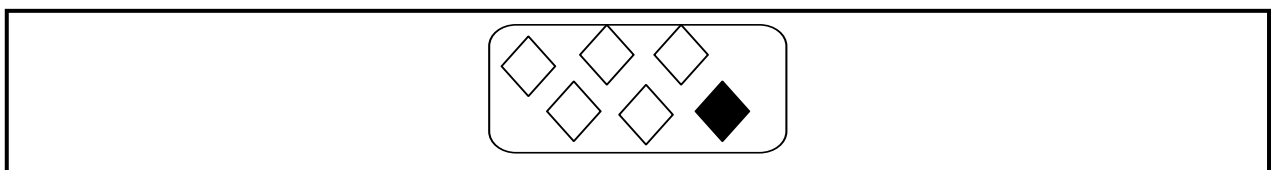


Figure 2: The length model of representing a fraction

Teachers were then asked:

- (i) Is there anything wrong with the representation? If yes, what would have been the correct label for the figure?
- (ii) What might have been the cause of his misconceived answer?
- (iii) What would you do to help him?

Observation Checklist. The 12 teachers that were interviewed were also observed in their natural classroom settings using an observation checklist. The checklist consists of six items representing pedagogical practices. Sample items are, “Create a conducive classroom environment”, “Focus on pupils understanding”, and “Make use of analogies and examples” among others. A tick (√) was made when there was a demonstrated evidence of the item and a cross (×) if otherwise.

Data Analysis

The instruments yielded both quantitative and qualitative data. The quantitative data were analysed using descriptive statistics with the help of Statistical Package for Service Solution (SPSS) software version 20. The data were summarised and transformed into frequencies, percentages, means, and standard deviations. The qualitative data were analysed thematically via the Atlas.ti version 7.5.18 software. The author independently read and re-read the qualitative data to systematically code, categorise, and interpret the data to provide clarifications (McMillan & Schumacher, 2006). Differences were reconciled and collapsed into themes (Ormston, Spencer, Barnard & Snape, 2013) consistent with the purpose of the study and the research questions.

RESULTS

The results of the study focused on primary school teachers’ knowledge and practice of teaching simple fractions in three domains as: teachers’ basic content knowledge, perceived pedagogical knowledge, and perceived pedagogical practice of teaching simple fractions.

1. Primary School Teachers’ Basic Content Knowledge of Simple Fractions

1.1. Quantitative Data

To assess the primary school teachers’ content knowledge base on simple fractions, 16 statements on basic fractional representations of which ten were correct (items 1, 2, 3, 7, 8, 10, 11, 12, 14, 16) and six (6) incorrect (4, 5, 6, 9, 13, and 15) were presented for the participants to indicate the extent to which they agreed or disagreed with items. Descriptive analysis of participants’ responses is presented in Table 1.

Table 1. Primary school teachers content knowledge on simple fractions

Items	Statement	Df (%)	Af (%)	M	SD
1	The part labeled A represents $1/8$	74 (49.3)	76 (50.7)	1.51	0.50
2	The part labeled H represents $3/16$	103 (68.7)	47 (31.3)	1.31	0.47
3	Part C + Part E = Part D	24 (16)	126 (84)	1.84	0.37
4	Line segment represents $0/4$	85 (56.7)	65 (43.3)	1.43	0.50
5	Line segment b represents $5/11$	84 (56)	66 (44)	1.44	0.50
6	Line segment c represents $2/5$	110 (73.3)	40 (26.7)	1.27	0.44
7	Line segment d represents $1/5$	109 (72.7)	41 (27.3)	1.27	0.45
8	The shaded portion in ‘a’ represents $1/2$	74 (49.3)	76 (50.7)	1.51	0.50

9	The shaded portion in 'b' is $\frac{1}{5}$	109 (72.7)	41 (27.3)	1.27	0.45
10	Fractions represent magnitude	83 (55.3)	67 (44.7)	1.45	0.50
11	Fractions are part of a whole	15 (10)	135 (90)	1.90	0.30
12	Fractions are part of a set	40 (26.7)	110 (73.3)	1.73	0.44
13	$\frac{99}{100}$ is less than $\frac{6}{7}$	100 (66.7)	50 (33.3)	1.33	0.47
14	Fractions are represented in multiple ways	32 (21.3)	118 (78.7)	1.79	0.41
15	Any fraction is greater than 1	81 (54)	69 (46.0)	1.46	0.50
16	Fractions have facts and procedures	24 (16)	126 (84)	1.84	0.37

Note: *SD*: Standard Deviation, *M*: Mean, *N*: Item Number, *D*: Disagree, *A*: Agree, *f*: Frequency

Table 1 indicates that participants' agreement to the items ranged from 23.7% to 90% while the disagreement ranged from 10% to 73.3% of the participants. Equal proportion of participants, 126 representing 84%, correctly agreed on items 3 (part-whole relation of fractions) and 16 (fractions have facts and procedures). This suggests that majority of the teachers have conceptual, procedural, and part-whole relation knowledge of basic fractions. This is followed by items 14 with 118 participants' representing 78.7% and item 12 of which 110 participants representing 73.3% correctly agreed to the statements. Similarly, 110 and 109 participants representing 73.3% and 72.7% correctly disagreed with items 6 and 9 respectively. Over 50% of the participants correctly disagreed on items 4, 5, 6, 13, and 15. Interestingly, although in items 2, 7 and 10 which are correct, 68.7%, 72.7% and 55.3% of the participants respectively incorrectly disagreed with the statements. The least incorrectly disagreed correct statement was on fractions as part of a whole (item 11) having a 10% of the participants while 31.3% the least agreed incorrect statement is on line segment representing $\frac{2}{5}$ (item 6) with 26.7% representation of the participants. Also, almost 45% of the participants incorrectly agreed with item 5 (44%) and item 4 (43.4%) suggesting that over 40% of primary school teachers are challenged with the line model representations of basic fractional concepts.

2. Primary School Teachers Perceived Pedagogical Knowledge on Simple Fractions

2.1 Quantitative Data

Eight items in the questionnaire (17-24) were designed to assess teachers' knowledge of pedagogical principles in teaching simple fractions. The descriptive analysis of participants' responses is presented in Table 2. The data indicate that over 75% of the teachers agreed correctly on all the pedagogical items for teaching fractions. The least agreed on item was 21 (knowledge of the learning theories and styles), recording 79.3% of the participants while the highest agreed item was 19 - creating connections of old and new lessons –recoded 90.70%. Equal proportions of teachers representing 88% each indicated they used fraction models and assessed pupils' performance in learning fractions. Slightly over 80% of the participants agreed that they had knowledge on how to engage pupils in hands-on experiences, diagnosed, and addressed pupils' misconceptions, as well as been aware that pupils come to the classroom with prior knowledge of fractions.

Table 2: Primary School

Items	Statement	Df (%)	Af (%)	M	SD
17	I encourage and motivate pupils	20 (13.30)	130 (86.70)	1.87	0.34
18	I use the fraction models	18 (12.00)	132 (88.00)	1.88	0.33
19	I create connections of old and new lessons	14(9.30)	136 (90.70)	1.91	0.29
20	I assess pupils' performance in fractions	18 (12.00)	132 (88.00)	1.88	0.33
21	I have knowledge of learning theories and styles	31 (20.7)	119 (79.3)	1.79	0.41
22	I know how to diagnose and address pupils' misconceptions in fractions	26 (17.3)	124 (82.7)	1.83	0.38

23	Pupils come into classrooms with prior knowledge in fractions	25 (16.7)	125 (83.3)	1.83	0.37
24	I know how to engage pupils in hands on experience	29 (19.3)	121 (80.7)	1.81	0.40

Teachers’ Perceived Pedagogical knowledge on Simple Fractions

Note: *SD*: Standard Deviation, *M*: Mean, *N*: Item Number, *D*: Disagree, *A*: Agree, *f*: Frequency

2.2. Qualitative Data

In responding to Question (i), Figure 1, six teachers correctly indicated the model was incorrect. Out of the six, two supported their responses with appropriate explanations. T6 explains as follows:

[ammm] to aid the child understand the set model I will make him understand that the denominator of the fraction normally represents the whole and the numerator represents the part that is to be taken from the whole. To interpret 1/5 with the set model I will let the child represent the denominator with five bottle tops out of which one will be taken out of the set to represent 1/5 of the whole.

In his response to (ii) of Figure 1, T7 said:

Ok maybe the cause is that when she was counting...she forgot to count one of them.

As to what he will do to help the child as in (iii) of Figure 1, he said:

Ok, what I will do is I will let the child to understand that [errrr] the numerator which is what five, the figures that are inside the box should be up to five after which you shade one out of the five would give you 1/5.

In response to Figure 2, eight out of the 12 teachers incorrectly saw nothing wrong with the length model and the rest admitted that there were errors. When those who indicated there was a problem with the representation were asked how they could help to correct it, T5 responded as follows:

To me personally, I think the right answer should have been $\frac{3}{4}$ and not $\frac{2}{4}$ because to get to 4 on the number line you will have to move through three steps thus from 2 to 3 and 4.

Some of the teachers expressed uncertainty in the representation. T2 for instance stated:

Maybe, looking at the figure, it represents whole numbers from zero to ten and should go chronologically. You cannot go and jump and start at 2. This is not a correct figure. That is my understanding. Maybe, if you start from 0 to 1, the answer will be and from 1 to 2, it is $\frac{2}{4}$. The $\frac{2}{4}$ comes from 2 to 0.

Interestingly, T1 felt that the representation was misleading indicating that:

[Ammm] to me I think the numbering on the number line is misleading and for that matter, it will be difficult for me to provide a befitting answer for the figure. The numbering should have being 0, $\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ up to $\frac{10}{4}$.

Asked how one could help the child on the length model, none of the teachers could advance convincing reasons to support their views. T2 for example explained:

Ok, I will let him or her understand that this is not a figure. When you draw shapes, for example, a triangle, a square when you divide it into four no matter the direction you should be able to indicate your 2/4 but as far as these numbers are concerned you have to start from zero, one, two to four. You don't have to start at the middle. For me, if you look at the way and manner in which he presented his answer it is 4/4 and not 2/4.

Similarly, T1 provided another confusing explanation based on tangible experiences of the child saying:

Ok [errrrrrr] using diagrams and then sharing among pupil for example, you draw a diagram, maybe having an orange you divide the orange into number of portions and then make the child know that maybe this orange has been divided into example eight portions which am going to share with two pupils in fractions this is how it will be. The two pupils that I am going to give to will represent the numerator and the total number of portions that we divided will be the denominator. Drawing it and showing them how to give to each person will help them know better on fractions.

From the interview data, of the five teachers (T1, T2, T5, T6, and T7) interviewed only one (T6) correctly knew the correct model and explained how the child could be assisted on the representation of 1/5. None of the teachers clearly knew what fraction is represented by the model and could provide an explanation to help in that situation.

3. Primary School Teachers' Perceived Pedagogical Practices in Teaching Simple Fractions

3.1 Quantitative Data

Six appropriate pedagogical items were listed for the teachers to indicate their level of agreement or disagreement in their practice. Table 3 presents data on the perceived pedagogical practices in teaching simple fractions. The data indicates that at least 87% of the participants agreed on all the listed pedagogical practices. Over 90% of the participants indicated they created a conducive classroom environment, diagnosed, and addressed pupils' misconceptions. The lowest percentage (87.3%) of participants indicated they focused on pupils understanding of concepts in practice. Item 30 (presenting lessons to meet the varied needs) had percentage of agreement (85.3%).

Table 3: Primary School Teachers' Perceived Pedagogical Practices in Teaching Simple Fractions

Items	Statement	Df (%)	Af (%)	M	SD
25	I teach from known to unknown	16 (10.70)	134 (89.30)	1.89	0.31
26	I focus much on pupils' understanding of concepts	19 (12.70)	131 (87.30)	1.87	0.33
27	I diagnose and address pupils' misconceptions	14 (9.30)	136 (90.70)	1.91	0.29
28	I create a conducive classroom environment	15 (10.00)	135 (90.00)	1.90	0.30
29	I make adequate use of manipulatives and representations	18 (12.00)	132 (88.00)	1.88	0.33
30	I present lessons to meet the varied needs of learners	22 (14.70)	128 (85.30)	1.85	0.35

Note: SD: Standard Deviation, M: Mean, N: Item Number, D: Disagree, A: Agree, f: Frequency

4. Primary school teachers' pedagogical practices in teaching simple fractions

To verify teachers' pedagogical practices in action, 12 of the participants were observed teaching simple fractions in their natural classrooms using an observation checklist. The

observed practices were tallied as frequency counts and presented in Table 4.

Table 4 shows that majority (66.7%) of the teachers presented their lessons from known to unknown and created conducive classroom atmospheres. The least demonstrated practice among teachers was presenting lessons to meet the varied needs of learners (25%), followed by focusing on pupils’ understanding (33.3). Furthermore, equal percentage of teachers (41.7%) made use of adequate manipulative materials and representations and analogies and examples in their practice.

Table 4: Primary school teachers’ pedagogical practices in teaching simple fractions

	Pedagogical Practice	T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	T11	T12	Total f(√)	% (√)
1	Teach from known to unknown	√	√	x	√	x	x	√	√	√	x	√	√	8	66.7
2	Focus much on pupils’ understanding	√	x	√	√	x	x	x	x	√	x	x	x	4	33.3
3	Create a conducive classroom environment	√	√	x	x	√	√	x	√	√	√	x	√	8	66.7
4	Make adequate use of manipulatives and representations	x	x	√	x	x	√	√	x	x	√	√	x	5	41.7
5	Make use of analogies and examples	√	x	√	x	√	x	x	x	√	x	x	√	5	41.7
6	Present lessons to meet the varied needs of learners	√	x	x	x	√	x	x	√	x	x	x	x	3	25.0
	Total	5	2	3	2	3	2	2	3	4	2	2	3		

5. Summary of Results

The present study explored teachers’ pedagogical content knowledge in the teaching of simple fractions in primary schools in one municipality in the Upper West Region of Ghana. The study revealed three main findings in respect to the research questions that guided the study.

First, results generally indicate that primary school teachers’ have limited content knowledge in simple fractions. Majority of the teachers either incorrectly disagreed with correct statements or incorrectly agreed with incorrect statements involving basic fractional concepts. They exhibited limited content knowledge in recognizing fractional representation as many could not identify and rectify children’s conceptual problems in misrepresentations of fractional concepts. Second, whereas the quantitative data suggested teachers to be knowledgeable in simple fractions, the qualitative data generated from the interviews suggested otherwise. Finally, most of the teachers perceived themselves to have the requisite pedagogical knowledge to teach simple fractions. However, the observational checklist data revealed that most teachers exhibited only two of the four pedagogical practices in their classrooms— teaching from known to unknown and creating a conducive classroom environment. Besides, the qualitative data suggests that most teachers could not translate perceived knowledge in their practice as most of them could not correctly explain how the child could be assisted with the misrepresentation of the two basic fractional concepts. Twenty-five to 58% of the teachers did not demonstrate four key pedagogical practices in their lessons namely: presenting lessons to meet the varied needs of learners, focusing much on pupils’ understanding, making adequate use of manipulatives and representations, and making use of analogies and examples.

DISCUSSIONS

The present study explored primary school teachers' pedagogical content knowledge-base and practices in teaching simple fractions. Overall, majority of the teachers perceived themselves to have content knowledge in conceptual, procedural and part-whole relation knowledge of basic fractions. Similarly, most of the teachers reported perceived pedagogical knowledge on all the pedagogical items for teaching fractions.

Notwithstanding the positive perceived content knowledge and pedagogical knowledge reported by the teachers, the study showed that they had limited content knowledge of basic fractional concepts. Most of the teachers incorrectly disagreed with correct statements and 26.7% - 46% incorrectly agreed with incorrect statements involving basic fractions (see Table 2). They were unable to assess the veracity or otherwise conceptual or procedural statements. For mathematics teachers to be able to react to pupils' ideas (Schifter, 2001), they must have clear understanding of basic concepts, properties, representations, and explanations (Samková, 2018; Shulmans, 1986; Torbeyns, et al., 2014).

Furthermore, the teachers had no clear understanding of the fractional representations as one-half and two-thirds of those interviewed could not see anything wrong with the incorrect set and line model representations of fractions respectively (see the qualitative data). This finding further confirms that even though the teachers were professionally trained and certificated to teach basic fractions in the curriculum using models and manipulative materials (MOE, 2012), their pedagogical content knowledge of fractions was limited. They lacked the capacity to react to pupils' misunderstandings of fractions. This knowledge gap might have been due to limitations in their professional preparation.

Another finding is that although the teachers perceived themselves to be pedagogically knowledgeable, over 50% of the those observed over relied on discussions and lecture methods with little use of manipulatives. They displayed limited use of models or manipulatives and varied presentations to meet learner needs contrary to curriculum recommendations. Many of the teachers demonstrated their lack of knowledge and skills to react to children's ideas (Schifter, 2001) in three ways. They were unable to: (i) diagnose the errors in pupils' representations, (ii) identify the possible causes of the misrepresentations and (iii) show how they can assist a child to unlearn and relearn correct representations. With no clear knowledge of the instructional representations that meet learner needs, learners' misconceptions and the causes of misconceptions or difficulties, teachers would not be able to effectively correct or probe pupils' responses. This notwithstanding, the primary school teachers demonstrated some pedagogical principles for teaching of simple fractions (Sibuyi, 2012), an indication that all is not lost in our bid to help the child learn fractions successfully.

These limitations, in general, may be largely due to the quality and depth of specific learning opportunities (Hattie, 2009) primary teacher education provides for mathematics teachers' professional development on teaching fractions. To address these recommendations, in-service training workshops and other continuous professional development opportunities with a focus on pedagogical content knowledge in fractions for teachers to increase their knowledge and practice in teaching simple fractions.

CONCLUSIONS

The findings of the present study revealed primary school teachers in one municipality in the Upper West Region possessed limited pedagogical content knowledge in the teaching of simple

fractions. Majority of teachers exhibited low content knowledge on the models of representing fractions, particularly in the length and set models of representing fractions. Similarly, majority of teachers demonstrated limited pedagogical knowledge on the teaching of simple fractions. This was evident from the teachers' inability to orally support and justify their responses. Finally, majority of teachers were reported to have displayed poor pedagogical practices in teaching simple fractions –majority resorted to the traditional lecture methods, made little use of manipulatives, and were unable to diagnose and address their pupils' misconceptions. To address primary school teachers' knowledge gap in fractions, we encourage institutions in charge of primary school mathematics teacher education to inject more of their energy into the development of teachers' pedagogical content knowledge in the teaching of fractions in practical ways that make sense to them.

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