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A NEW PROCESS FOR SOLVING LINEAR FRACTIONAL PROGRAMMING PROBLEM

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Abstract

There is a need for generalizing the simplex technique for linear programming to the ratio of linear functions or the ratio of quadratic functions and in such a situation, all the problems are fragments of a general class of optimization problems, termed in the literature as fractional programming problems. The linear fractional programming problem arise when there appears a necessity to optimize the efficiency in other activities also, for example, profit gained by company per unit of expenditure of labor, cost of production per unit of produced goods etc. Nowadays, because of deficit of natural resources, the use of such specific criteria becomes more and more topical and relevant. Linear Fractional problems arises in management decision making, research, finance, production, Health care and Hospital Planning and transportation etc., In this paper, a new process has been introduced for solving linear fractional programming problem. The main purpose of this method is to reduce the number of iterations in order to save time. By using the numerical examples, we suggest that the optimality of this method arises in a few numbers of iteration than the existing Simplex method for solving linear fractional programming problem. Also, in certain cases, the same number of iterations is maintained for the optimality.

Keywords*: Basic feasible solution, optimum solution, alternative simplex method, linear programming problem, linear fractional programming problem.*

INTRODUCTION

Linear-fractional programming (LFP) is a generalization of linear programming (LP) in which the objective function in a linear program is a linear function whereas the objective function in a linear-fractional program is a ratio of two linear functions. This field of LFP was developed by Hungarian mathematician B. Matros in 1960. Linear Fractional Programming Problems is studied by many authors charnes et al.(1962) reformulate Linear Fractional Programming Problem into a Linear Programming Problem. Among them M. Karade and P.G.Kumbhare gives the alternative approach for the simplex method (1980).Linear fraction programs and Singh C.(1981) discussed a useful study about the optimality condition in Fractional Programming. An integer solution Fractional Programming Problem was discussed by S.C.Sharma (2011) . Furthermore, Many researchers discussed about the alternative solutions of simplex method. Optimum solution of simplex method and Conventional Simplex Method, An Alternative Approach was given by N.W.Khobragade.(2013). Sophia Porchelvi R and Anna Sheela A introduced a new method for Linear Multi – Objective Fractional Transportation Problem.

This paper is organized as follows. In section 2, Mathematical formulation for linear fractional programming problem is given along with some basic definitions. An algorithm for solving linear fractional programming problem using conventional simple method is developed in section 3 and it is verified by the numerical example. The last section draws some concluding remarks.

MATHEMATICAL FORMULATION

1 The general linear programming problem in the form

$Maximixe\ Z = c_1 x_1 + c_2 x_2 + \ldots + c_n x_n$

Subject to the constraints

$$a_{i1} x_1 + a_{i2} x_2 + \ldots + a_{in} x_n = b_i$$
$$x_1, x_2, \ldots, x_n \geq 0$$

2 A linear fractional programming problem:

$Maximize\ Z = \frac{c\,x + \alpha}{d\,x + \beta}$

Subject to the constraints

$Ax \leq b,\ x \geq 0$.

Where i) A is an n × m matrix , A = [a_{ij}] (i = 1,…,m ; j = 1,…n)

ii) x, c and d are (n x 1) vectors , b is an (m x 1) vectors and

iii) α, β are scalars.

DEFINITIONS

(i) Degenerate Solution

A basic solution to the system is called Degenerate, if one or more of the basic variables vanish.

(ii) Basic Feasible Solution

A feasible solution to a Linear Programming Problem, which is also a basic solution to the problem is called a Basic Feasible Solution.

(iii) Optimal Solution

A basic feasible solution x_B to the Linear Programming Problem

Maximize z = c x

Subject to the constraints: A x ≤ b, x ≥ 0, is said to be an Optimal Solution, if all $c_j - z_j \leq 0$.

NEW ALGORITHM FOR SOLVING LINEAR FRACTIONAL PROGRAMMING PROBLEM

STEP 1:

First, observes whether all the right-side constants of the constraints are non-negative. If not, it can be changed into positive value on multiplying both the sides of the constraints by (-1).

STEP 2:

Converts the inequality constraints to equations by introducing the non-negative slack or surplus variables. The coefficients of slack or surplus variables are always taken zero in the objective function.

STEP 3:

Let x_B be the initial basic feasible solution of the given problem such that

$B x_B = b$

$x_B = b B^{-1}$ where $B = (b_1, b_2, \ldots, b_r, b_s, \ldots, b_m)$.

Further suppose that, $Z_1 = c_B x_B + \alpha$ and $Z_2 = d_B x_B + \beta$

Where c_B and d_B are the vectors having their components as the coefficients associated with the basic variables in the numerator and denominator of the objective function respectively.

STEP 4:

Compute the evaluations $c_j - z_j$ (j=1,2,……n) by using the relation

$c_j - z_j = c_j - c_B y_j$, Where $Y_j = B^{-1} a_j$.

Also compute $\frac{C_j - Z_j}{\Sigma y_j}$

Case (i) If $y_{ij} > 0$, then Go to Step 6

Case (ii) If $y_{ij} < 0$, then Go to Step 6

Case (iii) If $y_{ij} = 0$, then that column is undefined and we don't consider that column for further process.

Similar, computation can be made for denominator also.

STEP 5:

Now, compute the net evaluation Δ_j for each variable x_j by the formula

$$\Delta_j = z_2\left(\frac{C\,j - Z\,j}{\Sigma\, y\, j}\right) - z_1\left(\frac{d\,j - Z\,j}{\Sigma\, y\, j}\right)$$

STEP 6:

Case (i) If all $\Delta_j \leq 0$, then the initial basic feasible solution X_B is an optimum basic feasible solution.

Case (ii) If at least one $\Delta_j > 0$, go to the next step.

STEP 7:

If there is more than one positive Δ_j, then choose the most positive of them. Let it be Δ_r for some j = r.

Case (i) If all $y_{ir} \leq 0$, (i = 1, 2.....m), then there is an unbounded solution to the given problem.

Case (ii) If at least one $y_{ir} > 0$, (i = 1, 2.....m), then the corresponding vector y_r enters the basis y_B.

STEP 8:

Compute the ratios $\left\{\frac{x_{B\,i}}{y_{i\,r}}, y_{i\,r} > 0,\ i = 1,2,\ldots\ldots..m\right\}$ and choose the minimum of them. Let the minimum of these ratios be $\frac{x_{B\,k}}{Y_{k\,r}}$. Then the vector y_k will leave the basis y_B. The common element $Y_{k\,r}$, which is in the k^{th} row and the r^{th} column is known as the leading element (or pivotal element) of the table.

STEP 9:

Convert the leading element to unity by dividing its row by the leading element itself and all other elements in its column to zeroes by making use of the relations:

$$Y_{ij} = y_{ij} - \frac{y_{k\,j}}{y_{k\,r}} y_{ir},\ \text{i=1,2,…,m,i≠k}$$

$$Y_{kj} = \frac{y_{k\,j}}{y_{k\,r}} \qquad \text{j=1, 2…n.}$$

Step 10:

Go to Step 4 and repeat the computational procedure until either an optimum solution is obtained or there is an unbounded solution.

NUMERICAL EXAMPLE

Find the integer solution of the following fractional linear programming problem:-

Maximize $z = \frac{2x_1+x_2}{3x_1+x_2+6}$

Subject to

$5x_1 + 3x_2 \leq 6$

$7x_1 + x_2 \leq 6$

$x_1, x_2 \geq 0$

SOLUTION After adding slack variables x_3 and x_4, the simplex table gives

Table 1

			c_j	2	1	0	0	Min. Ratio
			d_j	3	1	0	0	θ
d_B	c_B	x_B	B	x_1	x_2	x_3	x_4	
0	0	x_3	6	5	3	1	0	2
0	0	x_4	6	7	1	0	1	6
			$\sum y_j$	12	4	1	1	
			$c_j - z_j$	2	1	0	0	
			$\frac{c_j - z_j}{\sum y_j}$	1/6	1/4	0	0	
			$d_j - z_j$	3	1	0	0	
			$\frac{d_j - z_j}{\sum y_j}$	1/4	1/4	0	0	
			Δ_j	1	3/2	0	0	

Therefore, x_1 enters into the basis and x_3 leaves the basis

Table 2

			c_j	2	1	0	0
			d_j	3	1	0	0
d_B	C_B	X_B	B	x_1	x_2	x_3	x_4
1	1	x_2	2	5 / 3	1	1/3	0
0	0	x_4	4	16 / 3	0	-1/3	1
		Δ_j	0	0	-	0	

Here $x_1 = 0, x_2 = 2$ and Z = ¼. Since all $\Delta_j \leq 0$, the current solution is optimal.

NUMERICAL EXAMPLE

Maximize $z = \frac{3x_1 + x_2}{2x_1 + 7}$

Subject to

$x_1 + 2x_2 \leq 3$

$3x_1 + 2x_2 \leq 6$

SOLUTION

TABLE 1

			c_j	6	5	0	0	Min. Ratio
			d_j	2	0	0	0	θ
d_B	c_B	x_B	B	x_1	x_2	x_3	x_4	
0	0	x_3	3	1	2	1	0	3
0	0	x_4	6	3	2	0	1	2
			$\sum y_j$	4	4	1	1	
			$c_j - z_j$	6	5	0	0	
			$\frac{c_j - z_j}{\sum y_j}$	3 / 2	5 / 4	0	0	
			$d_j - z_j$	2	0	0	0	
			$\frac{d_j - z_j}{\sum y_j}$	1 / 3	0	0	0	
			Δ_j	21 / 2	35 / 4	0	0	

Here, x_1 enters into the basis and x_4 leaves the basis.

TABLE 2

			c_j	6	5	0	0	Min. Ratio
			d_j	2	0	0	0	θ
d_B	c_B	x_B	B	x_1	x_2	x_3	x_4	
0	0	x_3	1	0	4 / 3	1	- 1 / 3	3 / 4
2	6	x_1	2	1	2 / 3	0	1 / 3	3
			Δ_j	0	27 / 2	0	-	

Therefore, x_2 enters into the basis and x_3 leaves the basis.

TABLE 3

			c_j	6	5	0	0
			d_j	2	0	0	0
d_B	c_B	x_B	B	x_1	x_2	x_3	x_4
0	5	x_2	3 / 4	0	1	3 / 4	-1 / 4
2	6	x_1	3 / 2	1	0	-1 / 2	1 / 2
			Δ_j	0	0	-81	-19

Here $x_1 = 3/2$, $x_2 = 3/4$ and $Z = Z_1 / Z_2 = 51/40$. Since all $\Delta_j \leq 0$, therefore the current solution is optimal.

CONCLUSION

In this paper, an alternative simplex method has been introduced for the solving linear fractional programming problem. By the numerical examples it has been verified and concluded that the iterations required for optimum solution of this method are less or almost equal as compared to the existing method for solving the linear fractional programming problem.

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